System Design Considerations for CAXSI: Coded Aperture X-ray Scatter Imaging

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CAXSI System Vision

Selected Key Components
- Distributed sources
  - Novel sources
  - Spectra
  - Primary aperture
- Various detectors
- Coded aperture(s)

![Graphs showing photon count vs. energy for different filters and materials.](image)
CAXSI System Vision

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Various detectors
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CAXSI System Vision

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Selected Key Ideas
- Physical modeling of signals
- Signature characterization
- System design motivated by integrated sensing and processing (compressive sensing)
- Integration of components
Selected Key Limitations
• Source spectral width
• Energy sensitivity of detectors
• Spatial extent of targets
• Unknown clutter in the luggage
• Low signal

Selected Key Ideas
• Design system to increase sensitivity and specificity → Overcome blurring effects, optimally measure photons
• Multifaceted design space
CAXSI Outline

• CAXSI System Vision

• Signature Analysis
  – Measurement space signature
  Forward models
  – Object space signature
  Reconstruction
  – Logical space signature
  SVD

• Conclusion
Signal Chain
Signature Definition

- Underlying characteristic of a target of interest under X-ray illumination
  - Employed to identify specific targets
  - Coherent scatter, incoherent scatter, attenuation
- Defined in three different spaces
  - Measured (Detector or measurement space)
  - Reconstructed (Target or object space)
  - Compressed (Logical or abstract space)
- Measured and reconstructed are acquired via experiments and/or MC simulations
- Compressed acquired via system design and integration
Example Signatures – Measurement Space (pencil beam, target alone)

- **Acrylic**
  - 150kVp, 0.1mm W
  - 150kVp, No Filter

- **Graphite**
  - 150kVp, 0.1mm W
  - 150kVp, No Filter
Example Signatures – Target Space
(150kVp, 0.1mm W)

Acrylic

NaCl

NH4NO3

Al

Milk chocolate

Graphite
Reconstructed signature with different spectra

- 150kVp, 0.1mm W
- 150kVp, No Filter
- 116kVp, 0.1mm W
- 116kVp, No Filter
Pencil Beam → Fanbeam Model

\[ \mathbf{r} = [x, y, 0], \quad \text{object point} \]
\[ \mathbf{r}' = [X_d, y', z'], \quad \text{detector point} \]
\[ \theta, \quad \text{scatter angle} \]
\[ \mathbf{s} = \mathbf{r}' - \mathbf{r}, \quad \text{scatter vector} \]
Physics-Based Model

- Based on a radiance model, propagated using ray projection
- Objects have scattering densities $f$ at each spatial location $\mathbf{r}$, as a function of momentum transfer $q$
- For coherent scatter at angle $\theta$, Bragg’s Law gives $q = 2k\sin(\theta/2)$
- Given vector $\mathbf{s}$ from scattering point to detector whose normal is $\mathbf{n}$, there is a geometric factor
  $\frac{|\mathbf{n} \cdot \mathbf{s}|}{s^2}$, where $\mathbf{s} = \mathbf{r}' - \mathbf{r}$
- Mask factor $T(\mathbf{r}, \mathbf{s})$
- Detector response $g(\mathbf{r}')$
in terms of impulse response

$$g(\mathbf{r}') = \int dA \int dq H'(\mathbf{r}', \mathbf{r}, q) f(\mathbf{r}, q)$$

$$H(\mathbf{r}', \mathbf{r}, q) = \frac{C}{dA} \frac{|\mathbf{n} \cdot \mathbf{s}|}{s^2} T(\mathbf{r}, \mathbf{s}) \left( \frac{1}{2q\sin\frac{\theta}{2}} \right) W\left( \frac{q}{2\sin\frac{\theta}{2}} \right)$$
Computation: Forward Model

\[ g(r') = C \int dr \left( \frac{n \cdot s}{s^2} \right) \cdot T(r, \hat{s}) \int dq \left( \frac{1}{2q \sin \frac{\theta}{2}} \right) \cdot W \left( \frac{q}{2 \sin \frac{\theta}{2}} \right) \cdot f(r, q) \]

- Detector response = Integrate object points × geometry factor × mask × integrate object momenta at scatter angle
- There exist opportunities to exploit symmetry
- Efficient computations have been implemented
- Backward model is the adjoint operator

Monte Carlo Pencil Beam Data of Al; Data, Model, Residual
Log-likelihood for Poisson Data

\[ g(r') = \int dA \int dqH(r', r, q) \, f(r, q) \rightarrow g = Hf \]

\[ g(m) = \sum_{i \in I} h(m, i) f(i) \]

- Forward model predicts the mean detector values
- A Poisson model is appropriate in many applications. Denote the random data by

\[ y(m) \sim \text{Poisson} \left( \sum_{i \in I} h(m, i) f(i) + \mu_b(m) \right), \quad m \in M \]

- The log-likelihood function for the data is

\[ l(y \mid f) = \sum_{m \in M} y(m) \ln \left( \sum_{i \in I} h(m, i) f(i) + \mu_b(m) \right) - \left( \sum_{i \in I} h(m, i) f(i) + \mu_b(m) \right) \]

where \( \mu_b(m) \) is the mean number of background counts
- Penalized ML estimation (also MAP); alternatively, variational Bayes (L. Carin, et al.)

\[ \hat{f}_{PML} = \arg \max_f l(y \mid f) - \beta \phi(f) \]
Pencil Beam Data, Forward Model and Monte Carlo

• Simulation parameters:
  – source to mask distance = 94.77 cm,
  – source to object distance = 57.78 cm,
  – source to detector distance = 109.47 cm.

• During reconstruction,
  – x resolution = 0.4 cm,
  – Number of pixels = 20,
  – momenta = 10:0.5:140,
  – downsampling factor = 1

• Simulated data: Al points at x = 9, 10, 11

• Monte Carlo data
Pencil Beam Data, Forward Model After 5 Iterations

- The true Al spectrum, and the spectral estimate at location 9, 10, 11.
- Ave. of the reconstructed object over the momentum transfer coordinate.
- Simulated detector data with Poisson noise (Maximum detector value set at 50).
- Estimated detector data.
- Absolute difference between the noisy simulated and estimated data.
Pencil Beam Data, Forward Model
After 200 Iterations

- The true Al spectrum, and the spectral estimate at location 9, 10, 11.
- Ave. of the reconstructed object over the momentum transfer coordinate.
- Simulated detector data with Poisson noise (Maximum detector value set at 50).
- Estimated detector data.
- Absolute difference between the noisy simulated and estimated data.
Pencil Beam Data, Monte Carlo After 5 Iterations

- The true Al spectrum, and the spectral estimate at location 8, 9.
- Ave. of the reconstructed object over the momentum transfer coordinate.
- The noisy Monte Carlo pencil beam data.
- Estimated detector data.
- Absolute difference between the noisy Monte Carlo and estimated data.
Pencil Beam Data, Monte Carlo After 200 Iterations

- The true Al spectrum, and the spectral estimate at location 8, 9.
- Ave. of the reconstructed object over the momentum transfer coordinate.
- The noisy Monte Carlo pencil beam data.
- Estimated detector data.
- Absolute difference between the noisy Monte Carlo and estimated data.
Target Signatures: Amorphous vs. Crystalline

Acrylic

Aluminum
Crystal system: Cubic
Crystal lattice: Face-centered
(Face-centered cubic, FCC)

Diffraction spectra are dependent on crystal structure
Classification based on materials crystallinity

Indexing texture of diffraction pattern may aid in fine-tuning material classification
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Design for Sampling Structure and Conditioning
Visibility in radius and angle
Singular Values
Pencil Beam, Coded Aperture

Periodic in phase code
Periodic in x code
Singular Vectors
Singular value analysis of coded aperture x-ray scatter imaging

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We examine the conditioning and singular value spectra of tomographic coded aperture scatter imagers. Scatter imaging may enable tomography of compact regions from snapshot measurements with singular values scaling favorably as compared to the Radon transform. The scaling of the singular value spectrum of the 2-D fan-beam geometry is confirmed through simulations. © 2012 Optical Society of America

OCIS codes: 110.6955, 110.7440.
Fig. 3. Singular value spectra for (a) \( L = 23 \) length quadratic residue code and \( L = 47 \) length quadratic residue code. The four curves indicate differing number of samples measured in the \( H \) (shift code) direction, and the \( V \) (scale code) direction.

<table>
<thead>
<tr>
<th>Illumination</th>
<th>CAXSI</th>
<th>Selected Volume</th>
<th>Radon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pencil</td>
<td>( \frac{\sqrt{N}}{\Omega} )</td>
<td>( \frac{1}{N} )</td>
<td>( \frac{1}{N} )</td>
</tr>
<tr>
<td>Plane</td>
<td>( \frac{1}{\Omega} )</td>
<td>( \frac{1}{N^2} )</td>
<td>( \frac{1}{N} )</td>
</tr>
<tr>
<td>Volume</td>
<td>( \frac{\Omega}{N^{3/2}} )</td>
<td>( \frac{1}{N^3} )</td>
<td>( \frac{1}{N} )</td>
</tr>
</tbody>
</table>
Multiple Source Illumination

Sensor sensitivity
Singular Values
Singular Vectors
Point Target Reconstruction
Multiple Points
SVD and Design

• Linear response functions map generalized measurement and include detector response, source structure, object basis (dictionaries)
• Restricted isometry, source similarity etc. can be analyzed
• Linear response guides design, feeds classification engines
• System response feeds adaptive structure
# Example Specifications: Knowledge-Enhance Compressive Measurements

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tunnel geometry</td>
<td>60 by 40 cm</td>
</tr>
<tr>
<td></td>
<td>10 by 10 cm?</td>
</tr>
<tr>
<td>Source(s)</td>
<td>1-4 sources, multifan collimation</td>
</tr>
<tr>
<td>Beam Energy</td>
<td>150-160 KV</td>
</tr>
<tr>
<td>Image resolution</td>
<td>1.5 mm cube</td>
</tr>
<tr>
<td>Momentum resolution</td>
<td>0.1 nm⁻¹</td>
</tr>
<tr>
<td>System volume</td>
<td>3.3 (L) by 1.3 (W) by 1.3 (H) meters</td>
</tr>
<tr>
<td>Pixel size</td>
<td>1 mm</td>
</tr>
<tr>
<td>Number of pixels</td>
<td>750 for attenuation signals</td>
</tr>
<tr>
<td></td>
<td>5,000 scatter pixels, including 128 energy resolving pixel.</td>
</tr>
</tbody>
</table>
KECoM AT

direct collimator and sensor array

scatter coded apertures and sensor arrays

x-ray source

x-ray source