Phys 5870: Modern Computational Methods in Solids
Homework 3

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We have chosen a basis for solving the Helium atom as:

$$\psi_1(\mathbf{x_1}, \mathbf{x_2}) = \frac{1}{\sqrt{2}} 1\mathrm{s}(\mathbf{r_1}) 1\mathrm{s}(\mathbf{r_2}) \left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right]$$

$$\psi_2(\mathbf{x_1}, \mathbf{x_2}) = \frac{1}{\sqrt{2}} \left[1\mathrm{s}(\mathbf{r_1}) 2\mathrm{s}(\mathbf{r_2}) - 1\mathrm{s}(\mathbf{r_2}) 2\mathrm{s}(\mathbf{r_1}) \right] |\uparrow\uparrow\rangle$$

$$\psi_3(\mathbf{x_1}, \mathbf{x_2}) = \frac{1}{2} \left[1\mathrm{s}(\mathbf{r_1}) 2\mathrm{s}(\mathbf{r_2}) - 1\mathrm{s}(\mathbf{r_2}) 2\mathrm{s}(\mathbf{r_1}) \right] \left[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right]$$

$$\psi_4(\mathbf{x_1}, \mathbf{x_2}) = \frac{1}{\sqrt{2}} \left[1\mathrm{s}(\mathbf{r_1}) 2\mathrm{s}(\mathbf{r_2}) - 1\mathrm{s}(\mathbf{r_2}) 2\mathrm{s}(\mathbf{r_1}) \right] |\downarrow\downarrow\rangle$$

$$\psi_5(\mathbf{x_1}, \mathbf{x_2}) = \frac{1}{2} \left[1\mathrm{s}(\mathbf{r_1}) 2\mathrm{s}(\mathbf{r_2}) + 1\mathrm{s}(\mathbf{r_2}) 2\mathrm{s}(\mathbf{r_1}) \right] \left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right]$$

Show that each of these states can be obtained as a Slater determinant, or a linear combination of two Slater determinants.